Springer

Journal of Mechanical Science and Technology

Journal of Mechanical Science and Technology 22 (2008) 1436~1443

www.springerlink.com/content/1738-494x

Numerical study on characteristics of turbulent two-phase gas-particle flow using multi-fluid model

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(Manuscript Received October 11, 2007; Revised April 11, 2008; Accepted April 15, 2008)

Abstract

The purpose of this research is to study numerically the turbulent gas-particle two-phase flow characteristics using the Eulerian-Eulerian method. A computer code is developed for the numerical study by using the k - ε - k_p two-phase turbulent model. The developed code is applied for particle-laden flows in which the particle volume fraction is between 10^{-5} and 10^{-2} for the Stokes numbers smaller than unity. The gas and particle velocities and the particle volume fraction obtained by using this code are in good agreement with those obtained by a commercial code for the gas-particle jet flows within a rectangular enclosure. The gas-particle jet injected into a vertical rectangular 3D enclosure is numerically modeled to study the effect of the Stokes number, the particle volume fraction and the particle Reynolds numbers. The numerical results show that the Stokes number and the particle volume fraction are important parameters in turbulent gas-particle flows. A small Stokes number ($St \leq 0.07$) implies that the particles are nearly at the velocity equilibrium with the gas phase, while a large Stokes number ($St \geq 0.07$) implies that the slip velocity between the gas and particle phase increases and the particle velocity is less affected by the gas phase. A large particle volume fraction ($\alpha_p \geq 0.0001$) implies that the effect of the particles on the gas phase momentum increases, while a small particle volume fraction ($\alpha_p \leq 0.0001$) implies that the particles would have no or small effect on the gas flow field. For fixed Stokes number and particle volume fraction, an increase of the particle Reynolds number results in a decrease of the slip velocity between the gas and particle velocities.

Keywords: Two-phase flow; Eulerian-Eulerian method; Stokes number; Particle volume fraction; Gas-particle jet

1. Introduction

Application of multi-phase flow appears in many industrial fields, such as the pulverized-coal furnace, oil burner nozzle, fluidized incinerator, bag filter and many kinds of dust collectors. Thus, many researchers have been involved in the field of multi-phase flow [1-4]. Gas-particle flow can be divided into two flow categories: dilute and dense flows. Crowe [5] concludes that dilute particle flow is controlled by the interaction of the surface force and volumetric force acting on the particle. On the other hand, the particle flow is controlled mainly by the collision between particles for dense flows.

The particle trajectory model is used for the two-phase flows with sparse and discontinuous particle phase and with large slip between the two phases [5]. On the other hand, the multi-fluid model can be used for the two phase flows with dense and continuum-like particle phase and with relatively small to moderate slip between gas and particle phases [6]. The particle trajectory model is frequently used for analyzing the gas-particle flow problems due to its easy implementation. In this model, the gas phase is considered as a continuum while the discontinuous particle phase is treated by the Lagrangian viewpoint. This method can be a good choice for a sparse particle flows, but it may not be a good choice for handling the two phase flows with dense particles where the number of parti-

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DOI 10.1007/s12206-008-0410-0

cles is very large. In the dense case, tracking the individual trajectory of each particle is very time consuming, and also the trajectories of the different particles may not show the proper particle velocity profile at some locations of interest [7]. To overcome this problem, the stochastic model is introduced to model the particle turbulence for the dense particle system [8].

The multi-fluid model assumes both the gas and particle phases to be a continuum. This model obtains physical quantities for particle flow from similar figures as gas governing equations [9-12]. The particle continuity equation in the multi-fluid model includes a term about particle dispersion effect [13-15]. Even though numerical analysis is unstable on large velocity difference between gas and particle phases, the particle dispersion effect is an important parameter in granular flows with large velocity slips between the two phases.

The effect of particle phase on the turbulent gas phase is important in modeling the multi-phase gas-particle flows. According to the turbulent intensity, multi-phase flow has different particle dispersion, effective viscosity of gas phase and drag force between gas and particles. Small sized particles tend to decrease the turbulent intensity, while large sized particles result in increased turbulent intensity.

The Stokes number ($St = \tau_R / \tau_F$) is an important parameter in multi-phase flow. This parameter is defined as the relation between the particle response time ($\tau_R = \rho_p d_p^2 / 18\mu$) and the flow time of gas phase ($\tau_F = L/u$). Here, u is the gas phase velocity and L is the distance scale. Particle velocity equilibrates gas velocity for small Stokes number ($St \ll 1$), but particles flow independently from gas velocity field for large Stokes number ($St \gg 1$). If the flow time of gas phase and the turbulence time scale are the same, a small Stokes number means that particles move in the same fashion as turbulent gas [16].

This study investigates effects of some important parameters such as Stokes number, particle volume fraction, particle Reynolds numbers on the gas-particle jet in a 3D rectangular enclosure by using the multi-fluid model called the k - ε - k_p model [17]. The range of the parameters studied in this paper is selected by considering the dilute flow found in pulverized coal furnaces with a typical particle volume fraction of $10^{-5} \sim 10^{-2}$ for a Stokes number of less than unity.

2. Computational method

2.1 Governing equations

The gas-particle two-phase flows with a Stokes number of less than unity can be expressed in the Eulerian-Eulerian form for both of the phases. Governing equations used in the multi-fluid model are expressed in the following general form.

$$\frac{\partial}{\partial t}(\rho\Phi) + \nabla \cdot \overrightarrow{J_\phi} = S_\phi + S_{r\phi} \tag{1}$$

$\overrightarrow{J_\phi}$ is the mass velocity of the physical quantity ϕ and is expressed as the sum of mass velocity ($\vec{F} = \rho\vec{V}$) by convection and mass velocity ($\Gamma_\phi \frac{\partial \phi}{\partial x_i}$) by diffusion. Tables 1 and 2 represent the gas and particle phases governing equations for a mono dispersed particle diameter. Table 3 shows the turbulent model coefficients. Various source terms are defined as

$$G_k = \mu_t \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \frac{\partial u_i}{\partial x_j} \tag{2}$$

$$G_p = \sum_p \sum_i \frac{\rho_p}{\tau_{rp}} \left[\left(c_p^k \sqrt{k k_p} - 2k \right) + \frac{\left(u_{pi} - u_i \right)}{n_p} \frac{\nu_p}{\sigma_p} \frac{\partial n_p}{\partial x_i} \right] \tag{3}$$

$$G_R = 2kS \tag{4}$$

$$G_{kp} = \mu_p \left(\frac{\partial u_{pj}}{\partial x_i} + \frac{\partial u_{pi}}{\partial x_j} \right) \frac{\partial u_{pi}}{\partial x_j} \tag{5}$$

Where u_i and u_{pi} are gas and particle velocities. ρ and ρ_p are gas density and particle apparent density. The turbulent gas viscosity μ_t, the particle phase viscosity μ_p and the particle response time for the mean motion τ_{rp} are defined as

$$\mu_t = \frac{C_\mu \rho k^2}{\varepsilon} \tag{6}$$

$$\mu_p = \rho_p v_p = \frac{C_{\mu p} \rho_p {k_p}^2}{\varepsilon_p} \tag{7}$$

$$\tau_{rp} = \frac{\overline{\rho_p} d_p^2}{18\mu} \left(1 + \frac{\mathrm{Re}_p^{2/3}}{6} \right)^{-1} \tag{8}$$

Table 1. Governing equations for gas-phase.

Equation	ϕ	Γ_ϕ	S_ϕ	$S_{r\phi}$
Continuity	1	0	0	$S=-\sum n_p \dot{m}_p$
Momentum	u_i	μ_e	$-\dfrac{\partial p}{\partial x_i}+\rho g_i+\dfrac{\partial}{\partial x_j}\left[\mu_e \dfrac{\partial u_j}{\partial x_i}\right]$	$u_i S+\sum \rho_p\left(u_{pi}-u_i\right)/\tau_{rp}$
Turbulent Kinetic Energy	k	$\dfrac{\mu_e}{\sigma_k}$	$G_k-\rho\varepsilon$	G_P+G_R
TKE Dissipation Rate	ε	$\dfrac{\mu_e}{\sigma_\varepsilon}$	$\dfrac{\varepsilon}{k}\left(C_{\varepsilon 1}G_k-C_{\varepsilon 2}\rho\varepsilon\right)$	$\dfrac{\varepsilon}{k}\left(C_{\varepsilon 1}G_P+C_{\varepsilon 1}G_R\right)$

Table 2. Governing equations for particle-phase.

Equation	ϕ	Γ_ϕ	S_ϕ	$S_{r\phi}$
Continuity	1	$\dfrac{v_p}{\sigma_p}$	0	0
Momentum	u_{pi}	$n_p v_p$	0	$n_p g_i+n_k\left(v_i-v_{pi}\right)\left(\dfrac{1}{\tau_{rp}}+\dfrac{\dot{m}_p}{m_p}\right)n_p+\dfrac{\partial}{\partial x_j}\left[n_p v_p\left(\dfrac{\partial v_{pj}}{\partial x_i}+\dfrac{\partial v_{pi}}{\partial x_j}\right)\right]+\dfrac{\partial}{\partial x_i}\left[\dfrac{v_p}{\sigma_p}\left(v_{pj}\dfrac{\partial n_p}{\partial x_i}+v_{pi}\dfrac{\partial n_p}{\partial x_j}\right)\right]$
Turbulent Kinetic Energy	k_p	$\dfrac{n_p v_p}{\sigma_p}$	0	$G_{kp}-\rho_p\varepsilon_p$
TKE Dissipation Rate	$\varepsilon_p=-2\left(\dfrac{m_p+\dot{m}_p\tau_{rp}}{m_p\tau_{rp}}\right)\times\left[\left(c_p^k\sqrt{kk_p}-k_p\right)+\dfrac{\left(u_i-u_{pi}\right)}{n_p}\dfrac{v_p}{\sigma_p}\dfrac{\partial n_p}{\partial x_i}\right]$			

Table 3. k - ε - k_p Turbulent model coefficients.

C_μ	σ_k	σ_ε	$C_{\varepsilon 1}$	$C_{\varepsilon 2}$	σ_p	c_p^k
0.09	1.0	1.3	1.44	1.92	0.7	0.75

where the particle Reynolds number is defined as $\mathrm{Re}_p=\left|u-u_p\right|d_p/\nu$ for particle diameter d_p, gas dynamic viscosity ν and gas phase viscosity μ. The effective gas phase viscosity is $\mu_e=\mu+\mu_t$.

m_p and $\dot{m}_p$ are the unit particle mass and the particle mass change rate. n_p and α_p are the particle number density and the particle volume fraction. g_i is the ith component of the acceleration of gravity.

2.2 System outline and computational grids

The two-phase flow system considered in this study is a rectangular enclosure with width [1m]× length [1m]× height [2m] as shown in Fig. 1. The homogeneous mixture of gas and particle flows into the system through a square port [0.2m×0.2m] at the center of bottom surface. The exit port of the enclosure extends over the top surface of the enclosure and the gas pressure within the enclosure is the same as the standard atmosphere. The properties of the gas and the particle are given in Table 4**.** The particles considered in this study are assumed to have a completely spherical shape with the same diameter.

Computational grids are constructed for the rectangular system as shown in Fig. 2 where the grids are constructed densely near the side walls, inlet port and outlet port. Grids are constructed in rectangular shapes with 38×38×42 in the x, y and z directions, which proved to be sufficient for obtaining numerically stable results in a grid independence test. Convergence criterion for the iterative calculation of a dependent variable ϕ is set as follows.

$$\text{Maximum relative error}=\max\left[\frac{\phi_{new}-\phi_{old}}{\phi_{new}}\right]<\alpha_a \tag{9}$$

Here, the maximum allowable relative error α_a is set to some different values between 10^{-3} and 10^{-4} for different variables for the iterative solution process.

Table 4. The properties of the gas and particles.

Gas density ρ [kg/m^3]	1.225
Gas viscosity μ [kg/m·s]	1.7894×10^{-5}
Particle density ρ_p [kg/m^3]	2,590

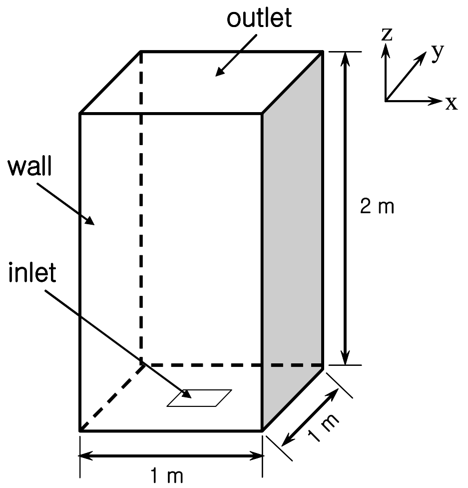


Fig. 1. Schematic diagram of the system.

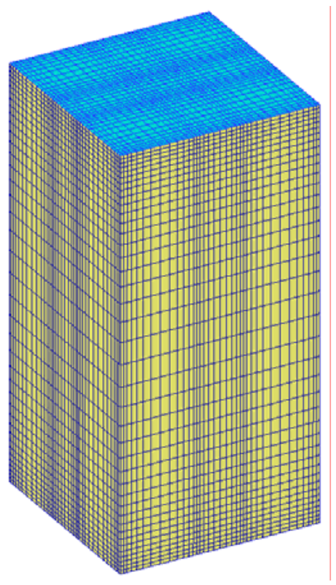


Fig. 2. Computational grid of the system.

2.3 Benchmarking of the code developed

The computer code developed in this study for the turbulent gas-particle two-phase flow is named as CCRHT3DNP. The code is benchmarked by comparing with the results obtained by using a widely used commercial code, FLUENT v. 6.2.16.

The CCRHT3DNP code is developed by using the k - ε - k_p model for turbulent gas-particle two-phase flows, while FLUENT has three different simple models for turbulent two-phase flow problems: mixture model, dispersed model and per phase model. For a benchmark test, the system and the computational grids explained in Fig. 1 are used for both codes. The particle volume fraction at the inlet port is set to be $\alpha_p = 0.0001$ and the gas-particle mass ratio is (ρ_p / ρ)=0.13. The gas and particle velocities at the inlet port are set equal to 1 [m/s], and the particle volume fraction at the inlet port is set to be $n_p = 8.304 \times 10^7$. The inlet port particle volume fraction considered here is selected by considering the mass ratio of coal and air in a typical pulverized-coal fired furnace where the mass ratio is within (12-15) percent coal and 1 percent air.

The results obtained by using CCRHT3DNP and FLUENT are compared to validate the CCRHT3DNP code developed in this study. Distributions of the gas and particle velocities and the particle volume fraction along the x-axis are compared at z=1.0 [m], y=0.5 [m]. Figs. 3 and 4 show that the gas and particle velocity distributions obtained from the two codes programmed with different mathematical models for the two-phase flow compared fairly well with each other. The model used in the CCRHT3DNP code is more elaborate than those used in the commercial code, FLUENT. This difference in mathematical models reveals that the velocity profiles obtained from the two codes show some differences in the shapes of velocity profiles, especially near the edge of the jet where the velocity profiles obtained from the CCRHT3DNP appear smoother distributions than those obtained from FLUENT. The results shown in Fig. 5 are the profiles of particle volume fraction obtained from the two different codes. From these comparisons, we conclude that the CCRHT3DNP code developed in this study can be used to study the characteristics of the turbulent two-phase flow jets.

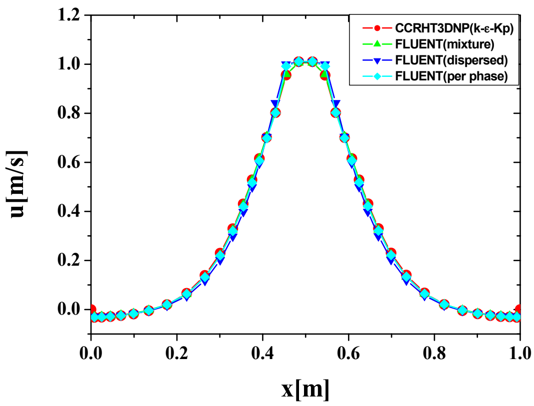


Fig. 3. Comparison of the gas velocity profiles obtained from different models.

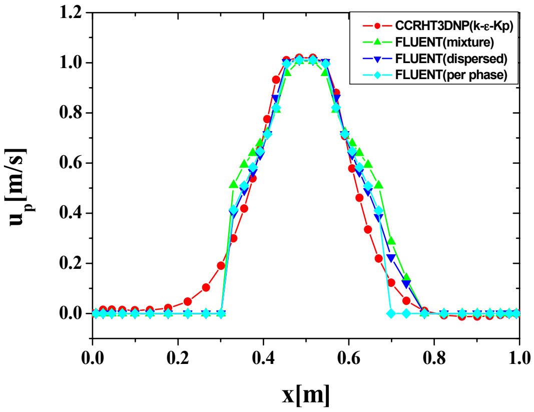


Fig. 4. Comparison of the particle velocity profiles obtained from different models.

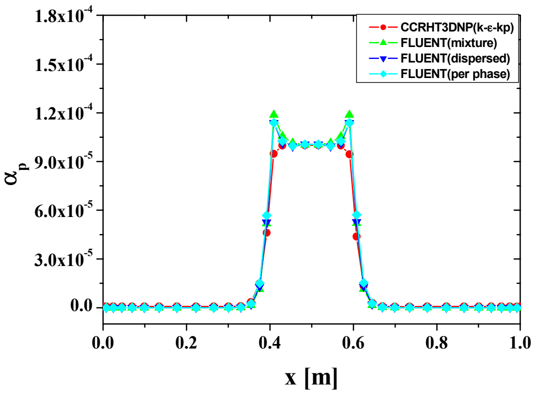


Fig. 5. Comparison of the particle volume fraction profiles obtained from different models.

3. Parametric studies of turbulent gas-particle jet

The Stokes number ($St = \tau_R / \tau_F$), which is an important parameter in two-phase flow, is defined as the ratio of particle response time ($\tau_R = \rho_p d_p^2 / 18\mu$) to the flow time ($\tau_F = L/u$). A large Stokes number implies that the particle phase flow is delayed as compared to the gas phase flow, while small Stokes number implies that the two-phases are in good coincidence showing small slip between the two-phases. The particle volume fraction ($\alpha_p = \rho_p / \overline{\rho_p}$) is defined as the ratio of the apparent particle density ($\rho_p = n_p m_p$) to the material density of the particle ($\overline{\rho_p}$). A large particle volume fraction implies that the particle number density is large and the collision effect between particles is more important. The gas Reynolds number ($\mathrm{Re} = uD/\nu$) is related to the gas flow time. A large gas phase Reynolds number implies that the gas velocity is large, and the flow time of gas phase is short. The particle Reynolds number ($\mathrm{Re}_p = |u - u_p| d_p / \nu$) is related to the velocity slip between the particle and the gas. Increase of the particle Reynolds number implies that the slip velocity between the particle and the gas is increased.

To study the effects of some important parameters on the two-phase jet flow, the CCRHT3DNP code developed in this study is used. The parameters considered for detailed study are the Stokes number, the particle volume fraction and the particle Reynolds number.

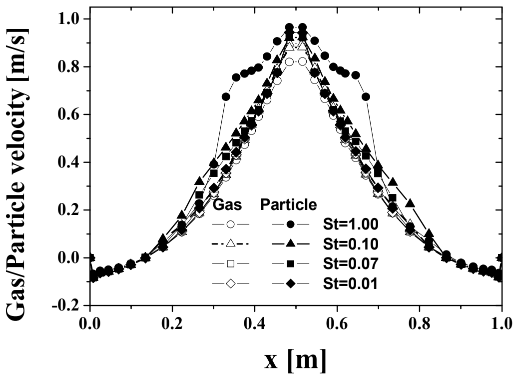
3.1 Effect of the stokes number

To study the effect of the Stokes number on the two-phase flow within the rectangular enclosure, the particle volume fraction at the inlet port is set equal to a constant value of 1.0×10^{-4}. The gas and particle velocities are set equal to a constant value of 1.0 [m/s], and therefore the gas and the particle Reynolds numbers at the inlet port are $\mathrm{Re} = 6.85 \times 10^4$ and $\mathrm{Re}_p = 0.0$. Four different values of the inlet port Stokes number between $0.01 \le St \le 1.0$ as shown in Table 5 are considered for this study.

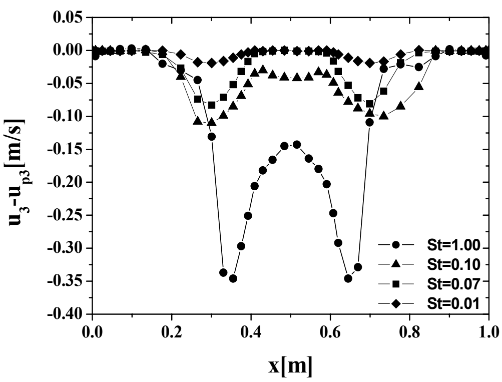
Fig. 6 shows the gas and particle phase velocity distributions along the x-axis at y=0.5 [m] and z=1.5 [m] for different Stokes numbers. For small Stokes numbers (St ≤ 0.07), the gas and particle phase velocities show nearly the same magnitude,

Table 5. Inlet port stokes numbers considered.

St	d_p [μm]	n_p [particles/m^3]
0.01	50	1.5×10^9
0.07	132	8.3×10^7
0.10	200	2.4×10^7
1	500	1.5×10^6



(a) Gas-particle velocities distribution



(b) Velocity slip between gas and particle

Fig. 6. Comparison of the gas and particle velocities at y=0.5 [m] and z=1.5 [m] for different inlet port Stokes numbers.

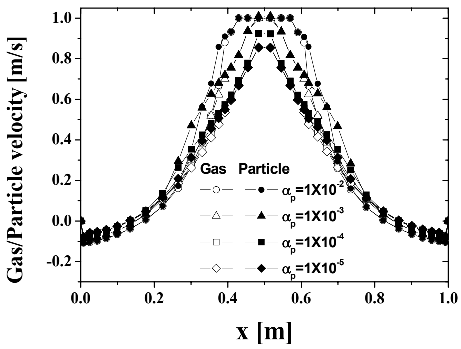
which implies that the velocity slip between the two phases is negligible. For $St = 0.07$, the velocity slip between the gas and the particle phases becomes recognizable near the edge of the jet flow. And for large Stokes numbers ($St \geq 0.07$), the particle velocities near the edge of the jet show larger values than the gas phase velocities, indicating large velocity slips between the two phases. Velocity slips between the two phases increase as the Stokes number increases. From this study, we may conclude that the velocity slip between the gas and the particle phases may be neglected for the Stokes number less than 0.07.

3.2 Effect of the particle volume fraction

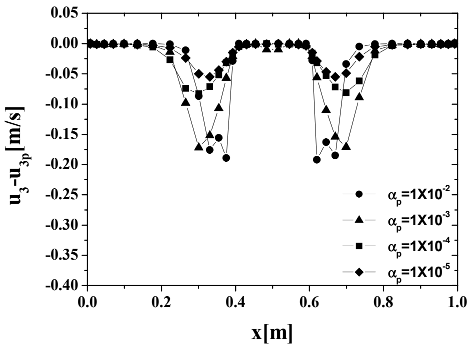
To study the effect of the particle volume fraction (α_p) on the gas-particle two-phase flow within the rectangular enclosure, the particle diameter, the Stokes number, the gas and particle Reynolds numbers are set equal to constant values of 132 [μm], 0.07, 6.85×10^4 , and 0. Four different values of the inlet port particle volume fraction between $1.0\times10^{-5} \leq \alpha_p \leq 1.0\times10^{-2}$ are considered for this study as summarized in Table 6.

Table 6. Inlet port particle volume fractions considered.

α_p	n_p [particles/m^3]
1.0×10^{-2}	8.30×10^9
1.0×10^{-3}	8.30×10^8
1.0×10^{-4}	8.30×10^7
1.0×10^{-5}	8.30×10^6



(a) Gas-particle velocities distribution



(b) Velocity slip between gas and particle

Fig. 7. Comparison of the gas and particle velocities at y=0.5 [m] and z=1.5 [m] for different inlet port particle volume fractions.

Table 7. Inlet port particle Reynolds numbers considered.

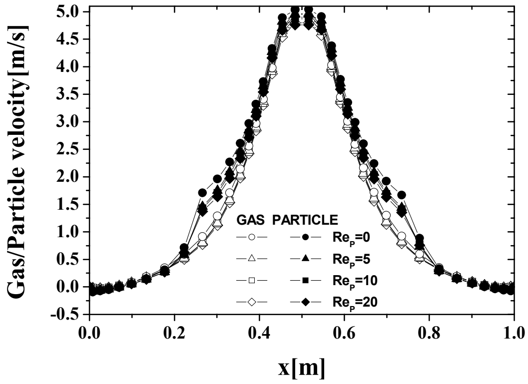
Re_p	u_p [m/s]
20	2.79
10	3.89
5	4.45
0	5.00

Fig. 7 shows the gas and particle phase velocity distributions along the x-axis at y=0.5 [m] and z=1.5 [m] for different inlet port particle volume fractions. Large particle volume fraction means that the density of the gas-particle mixture is large. For $\alpha_p \leq 1.0\times10^{-4}$, the two phase flow behaves such as a single gas phase flow and the particle phase shows a negligible effect on the overall jet flow. However, for $\alpha_p \geq 1.0\times10^{-4}$, the particle phase begins to affect the gas and particle velocity distributions by increasing the velocities of both phases over the whole jet region.

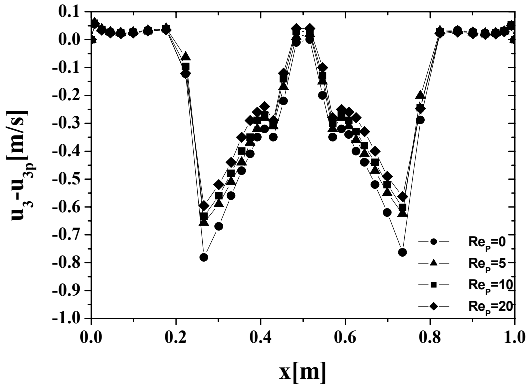
3.3 Effect of the particle Reynolds number

The particle Reynolds number is defined by using the slip velocity, $\left|u-u_p\right|$, between the gas and particle phases. To study the effect of the inlet port particle Reynolds number on the two-phase flow within the rectangular enclosure, the gas velocity, the Stokes number and the particle volume fraction at the inlet port are set equal to constant values of 5 [m/s], 0.35 and 1.0×10^{-4}. Four different values of the inlet port particle Reynolds number between $0 \leq Re_p \leq 20$ as summarized in Table 7 are considered for numerical study.

Fig. 8 shows the gas and particle phase velocity distributions along the x-axis at y=0.5 [m] and z=1.5 [m] for different inlet port particle Reynolds numbers. The gas and particle phases are easy to keep the velocity equilibrium in the gas phase dominant flow. The particle phase velocity is much larger than the gas phase velocity near the edge of jet since the particle phase is accelerated by the gas phase and the velocity slip between the two phases appears to be significant near the edge of jet. For fixed Stokes number and particle volume fraction, the increase in the particle Reynolds number results in slight decrease in the velocity slip between the two phases, which is mainly due to the increase of gas and particle interactions.



(a) Gas-particle velocities distribution



(b) Velocity slip between gas and particle

Fig. 8. Comparison of the gas and particle velocities for different inlet port particle Reynolds numbers at y=0.5 [m] and z=1.5 [m].

4. Conclusions

This study is focused on developing a computer code that can be used to analyze the turbulent gas-particle two-phase flows using the Eulerian-Eulerian method called the k - ε - k_p two-phase turbulent model. The code is then applied for analysis of a particle-laden air jet into a rectangular box with the particle volume fraction between 10^{-5} and 10^{-2} and with the Stokes number less than unity. The results obtained by using the code developed here show fairly good agreement with those obtained by a commercial code.

The effects of the Stokes number, the inlet port particle volume fraction, the gas and particle Reynolds number on the turbulent two-phase flow characteristics of the gas-particle jet are also studied by using the computer code developed in this study. The numerical results show that the Stokes number is an important parameter in determining the magnitude of the velocity slip between the gas and particle phase, and it is found that for $St \gg 0.07$ the velocity slip becomes more important. The study also shows that the inlet port particle volume fraction affects the momentum of both phases, and for large particle volume fraction ($n_p \geq 0.0001$) the effect of the particle phase on the gas phase momentum becomes recognizable. For fixed Stokes number and particle volume fraction, increased particle Reynolds number results in a decrease in the velocity slip due to the increased interaction between the two phases.

Acknowledgment

Authors wish to acknowledge the financial support from the ChungAng University and the CERC (Combustion Engineering Research Center).

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